DOUBLEROOT

Cheat Sheet – Complex Numbers

Definition

$$z = x + iy$$

$i = \sqrt{-1};\ x = \mathrm{Re}(z);\ y = \mathrm{Im}(z)$

Algebra

$$z_1 = x_1 + iy_1;\ z_2 = x_2 + iy_2$$

$$z_1 \pm z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_2 y_1 + x_1 y_2)$$

$$\frac{z_1}{z_2} = \frac{(x_1 x_2 + y_1 y_2) + i(x_2 y_1 - y_2 x_1)}{x_2^2 + y_2^2}$$

Modulus: |z|

$$z = x + iy \Rightarrow |z| = \sqrt{x^2 + y^2}$$

Properties

$$|z| = |-z| = |\bar{z}| = |-\bar{z}|$$

$$|z| = \pm\mathrm{Re}(z) \Leftrightarrow \mathrm{Im}(z) = 0$$

$$|z| = \pm\mathrm{Im}(z) \Leftrightarrow \mathrm{Re}(z) = 0$$

$$|z_1 z_2| = |z_1||z_2|$$

$$|z^n| = |z|^n$$

$$|z_1/z_2| = |z_1|/|z_2|$$

$$|z_1 + z_2| \le |z_1| + |z_2|$$

Equality holds when $0, z_1$, and z_2 are collinear and z_1 and z_2 are on the same side of 0

$$|z_1 - z_2| \ge \big||z_1| - |z_2|\big|$$

Equality holds when $0, z_1$, and z_2 are collinear and z_1 and z_2 are on the same side of 0

Conjugate: $\bar{z}$

$$z = x + iy \Rightarrow \bar{z} = x - iy$$

Properties

$$\bar{\bar{z}} = z$$

$$\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$$

$$\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$$

$$\overline{(z^n)} = (\bar{z})^n$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

$$|\bar{z}| = |z|$$

$$\arg(\bar{z}) = \underset{k \in Z}{2k\pi} - \arg(z)$$

$$\bar{z} = z \Leftrightarrow \mathrm{Im}(z) = 0$$

$$\bar{z} = -z \Leftrightarrow \mathrm{Re}(z) = 0$$

Argument: arg(z)

$$z = x + iy \Rightarrow \arg(z) = \theta, \text{where } \tan\theta = y/x$$

Quadrant of θ is determined by signs of x and y

Properties

$$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$$

$$\arg(z^n) = n\arg(z)$$

$$\arg(z_1/z_2) = \arg(z_1) - \arg(z_2)$$

$$\arg(z) = \underset{k \in Z}{2k\pi} \Leftrightarrow \mathrm{Im}(z) = 0$$

$$\arg(z) = \underset{k \in Z}{k\pi} + \pi/2 \Leftrightarrow \mathrm{Re}(z) = 0$$

Polar Form

$$z = |z|e^{i\theta} = |z|(\cos\theta + i\sin\theta)$$

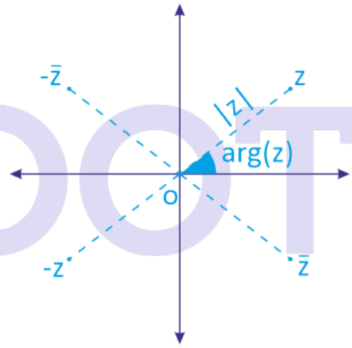
$\theta = \arg(z)$

De Moivre's Formula

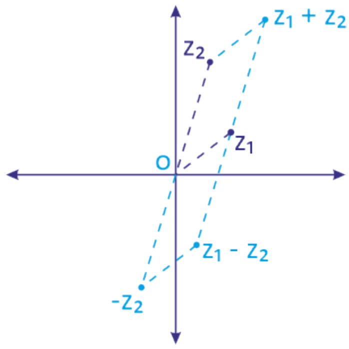
$$(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$$

Geometry

Modulus
Conjugate
Argument



Sum
Difference



Product
Quotient