

Algebra

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a - b)(a + b)$

4. $ax^2 + bx + c =$
 $a \left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right),$
if $b^2 - 4ac \geq 0$

Trigonometry

Definitions

1. $\tan x = \frac{\sin x}{\cos x}$
2. $\cot x = \frac{\cos x}{\sin x}$
3. $\sec x = \frac{1}{\cos x}$
4. $\csc x = \frac{1}{\sin x}$

Identities

1. $\cos^2 x + \sin^2 x = 1$
2. $\sin 2x = 2 \sin x \cos x$
3. $\cos 2x =$
 $= \cos^2 x - \sin^2 x$
 $= 2 \cos^2 x - 1$
 $= 1 - 2 \sin^2 x$
4. $\tan^2 x + 1 = \sec^2 x$
5. $\cot^2 x + 1 = \csc^2 x$
6. $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
7. $\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$

Derivatives

1. $(\sin x)' = \cos x$
2. $(\cos x)' = -\sin x$
3. $(\tan x)' = \sec^2 x$
4. $(\cot x)' = -\csc^2 x$
5. $(\sec x)' = \tan x \sec x$
6. $(\csc x)' = -\cot x \csc x$
7. $(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}}$
8. $(\arccos x)' = \frac{-1}{\sqrt{1 - x^2}}$
9. $(\arctan x)' = \frac{1}{1 + x^2}$

Integrals

1. $\int \cos x \, dx = \sin x$
2. $\int \sin x \, dx = -\cos x$
3. $\int \sec^2 x \, dx = \tan x$
4. $\int \csc^2 x \, dx = -\cot x$
5. $\int \cot x \, dx = \ln |\sin x|$
6. $\int \tan x \, dx =$
 $= -\ln |\cos x|$
7. $\int \sec x \, dx =$
 $= \ln |\tan x + \sec x|$
8. $\int \csc x \, dx =$
 $= -\ln |\cot x + \csc x|$

Hyperbolic functions

Definitions

1. $\sinh x = \frac{e^x - e^{-x}}{2}$
2. $\cosh x = \frac{e^x + e^{-x}}{2}$
3. $\operatorname{csch} x = \frac{1}{\sinh x}$
4. $\operatorname{sech} x = \frac{1}{\cosh x}$
5. $\tanh x = \frac{\sinh x}{\cosh x}$
6. $\coth x = \frac{\cosh x}{\sinh x}$

Identities

1. $\cosh^2 x - \sinh^2 x = 1$
2. $1 - \tanh^2 x = \operatorname{sech}^2 x$
3. $\coth^2 x - 1 = \operatorname{csch}^2 x$
4. $\sinh 2x = 2 \sinh x \cosh x$
5. $\cosh 2x =$
 $= \cosh^2 x + \sinh^2 x$
 $= 2 \cosh^2 x - 1$
 $= 2 \sinh^2 x + 1$

Derivatives

1. $(\sinh x)' = \cosh x$
2. $(\cosh x)' = \sinh x$
3. $(\tanh x)' = \operatorname{sech}^2 x$
4. $(\coth x)' = -\operatorname{csch}^2 x$
5. $(\operatorname{sech} x)' =$
 $= -\tanh x \operatorname{sech} x$
6. $(\operatorname{csch} x)' =$
 $= -\coth x \operatorname{csch} x$
7. $(\operatorname{arcsinh} x)' =$
 $= \frac{1}{\sqrt{x^2 + 1}}$
8. $(\operatorname{arccosh} x)' =$
 $= \frac{1}{\sqrt{x^2 - 1}}$
9. $(\operatorname{arctanh} x)' =$
 $= \frac{1}{x^2 - 1}$

Symmetry & Periodicity

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|---------------------------------|---------------------------------|----------------------------------|
| 1. $\sin(-x) = -\sin(x)$ | 5. $\cos(-x) = \cos(x)$ | 9. $\tan(-x) = -\tan(x)$ |
| 2. $\sin(x + \pi/2) = \cos(x)$ | 6. $\cos(x + \pi/2) = -\sin(x)$ | 10. $\tan(x + \pi/2) = -\cot(x)$ |
| 3. $\sin(x - \pi/2) = -\cos(x)$ | 7. $\cos(x - \pi/2) = \sin(x)$ | 11. $\tan(x - \pi/2) = -\cot(x)$ |
| 4. $\sin(x \pm \pi) = -\sin(x)$ | 8. $\cos(x \pm \pi) = -\cos(x)$ | 12. $\tan(x \pm \pi) = \tan(x)$ |

x	$-\pi$	$-\pi/2$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π	$3\pi/2$	2π
$\sin(x)$	0	-1	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	-1	0
$\cos(x)$	-1	0	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\tan(x)$	0	-	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	0	-	-
$\cot(x)$	-	0	-	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	-	0	1

Powers, Exponents & Logarithms

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|--------------------------------|--|---|
| 1. $x^{a+b} = x^a \cdot x^b$ | 6. $e^x : (-\infty, \infty) \rightarrow (0, \infty)$ | 9. $\ln(x) : (0, \infty) \rightarrow (-\infty, \infty)$ |
| 2. $x^{a-b} = \frac{x^a}{x^b}$ | | 10. $\ln(a \cdot b) = \ln(a) + \ln(b)$ |
| 3. $x^{-a} = \frac{1}{x^a}$ | 7. $e^{\ln(x)} = x$ | 11. $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$ |
| 4. $x^{a \cdot b} = (x^a)^b$ | | 12. $\ln(a^b) = b \ln(a)$ |
| 5. $x^{a/b} = \sqrt[b]{x^a}$ | 8. $\ln(e^x) = x$ | |

Derivatives & Integrals

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|---|--|------------------------------------|
| 1. $(x^n)' = nx^{n-1}$ | 3. $(e^{ax})' = ae^{ax}$ | 5. $(\ln(x))' = \frac{1}{x}$ |
| | | 6. $\int \frac{dx}{x} = \ln(x)$ |
| 2. $\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$ | 4. $\int e^{ax} dx = \frac{e^{ax}}{a}$ | 7. $\int \ln(x) dx = x \ln(x) - x$ |

Other Integrals

- $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan \frac{x}{|a|}$
- $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln |x + \sqrt{x^2 \pm a^2}|$

Inverse trigonometric functions

$$\arcsin x = \arccos \sqrt{1 - x^2}$$

	$y = \arcsin(x)$	$y = \arccos(x)$	$y = \arctan(x)$	$y = \operatorname{arccot}(x)$
$\sin(y)$	x	$\sqrt{1 - x^2}$	$\frac{x}{\sqrt{x^2 - 1}}$	$\frac{1}{\sqrt{1 + x^2}}$
$\cos(y)$	$\sqrt{1 - x^2}$	x	$\frac{1}{\sqrt{1 + x^2}}$	$\frac{x}{\sqrt{x^2 - 1}}$
$\tan(y)$	$\frac{x}{\sqrt{1 - x^2}}$	$\frac{\sqrt{1 - x^2}}{x}$	x	$\frac{1}{x}$
$\cot(y)$	$\frac{\sqrt{1 - x^2}}{x}$	$\frac{x}{\sqrt{1 - x^2}}$	$\frac{1}{x}$	x

Inverse hyperbolic functions

	$y = \operatorname{arcsinh}(x)$	$y = \operatorname{arccosh}(x)$	$y = \operatorname{arctanh}(x)$	$y = \operatorname{arcoth}(x)$
$\sinh(y)$	x	$\sqrt{x^2 - 1}$	$\frac{x}{\sqrt{1 - x^2}}$	$\frac{\operatorname{sign} x}{\sqrt{x^2 - 1}}$
$\cosh(y)$	$\sqrt{x^2 + 1}$	x	$\frac{1}{\sqrt{1 - x^2}}$	$\frac{ x }{\sqrt{x^2 - 1}}$
$\tanh(y)$	$\frac{x}{\sqrt{x^2 + 1}}$	$\frac{\sqrt{x^2 - 1}}{x}$	x	$\frac{1}{x}$
$\coth(y)$	$\frac{\sqrt{x^2 + 1}}{x}$	$\frac{x}{\sqrt{x^2 - 1}}$	$\frac{1}{x}$	x