Table of Basic Integrals

$\int 0 \,\mathrm{d}x = C,$ $\quad x \in (-\infty, \infty)$

$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C,$ $\quad x \in (-\infty, \infty),$ for n whole number, $n \geq 0$

$x \in (-\infty, 0), \quad x \in (0, \infty)$ for n whole number, $n < 0,\ n \neq -1$

for n rational, $n \neq -1$ see solved examples.

$\int \sin x \,\mathrm{d}x = -\cos x + C,$ $\quad x \in (-\infty, \infty)$

$\int \cos x \,\mathrm{d}x = \sin x + C,$ $\quad x \in (-\infty, \infty)$

$\int \frac{1}{\cos^2 x} \,\mathrm{d}x = \mathrm{tg}\, x + C,$ $\quad x \in (-\pi/2 + k\pi, \pi/2 + k\pi), k$ whole number

$\int \frac{1}{\sin^2 x} \,\mathrm{d}x = -\mathrm{cotg}\, x + C,$ $\quad x \in \left(k\pi, (k+1)\pi\right), k$ whole number, $k \in \left(0, \infty\right)$

$\int \frac{1}{\sqrt{1-x^2}} \,\mathrm{d}x = \arcsin x + C,$ $\quad x \in \left(-1, 1\right)$

$\int \frac{1}{\sqrt{1-x^2}} \,\mathrm{d}x = -\arccos x + C,$ $\quad x \in \left(-1, 1\right)$

$\int \frac{1}{1+x^2} \,\mathrm{d}x = \mathrm{arctg}\, x + C,$ $\quad x \in \left(-\infty, \infty\right)$

$\int \frac{1}{1+x^2} \,\mathrm{d}x = -\mathrm{arccotg}\, x + C,$ $\quad x \in \left(-\infty, \infty\right)$

$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C,$ $\quad x \in \left(-\infty, \infty\right)$

$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C,$ $\quad x \in \left(-\infty, \infty\right)$ (for $a > 0,\ a \neq 1$)

$\int \frac{1}{x} \,\mathrm{d}x = \ln|x| + C,$ $\quad x \in \left(-\infty, 0\right), x \in \left(0, \infty\right)$

Substitution

$\boxed{ax + b = t}$

$$\int f(ax+b) \,\mathrm{d}x = \frac{1}{a} F(ax+b) + C$$

$\boxed{f(x) = t}$

$$\int \frac{f'(x)}{f(x)} \,\mathrm{d}x = \ln\left|f(x)\right| + C$$

Volume of Rotational Body

$$V = \pi \int_a^b \left(f(x)\right)^2 \,\mathrm{d}x$$

Length of Curve

$$l = \int_a^b \sqrt{1 + [f'(x)]^2} \,\mathrm{d}x$$

Surface of Rotational Body

$$S = 2\pi \int_a^b f(x) \sqrt{1 + \left[f'(x)\right]^2} \,\mathrm{d}x$$

Area between non negative function *f* and x-axis for a parametrically defined function

$$\left.\begin{matrix} x = \varphi(t) \\ y = \psi(t) \end{matrix}\right\} t \in \langle \alpha, \beta \rangle,\ y = f(x),\ a = \varphi(\alpha),\ b = \varphi(\beta).$$

$$S = \int_a^b f(x) \,\mathrm{d}x = \left| \begin{matrix} x = \varphi(t) \\ \mathrm{d}x = \varphi'(t) \,\mathrm{d}t \end{matrix} \right| = \int_\alpha^\beta \psi(t) \varphi'(t) \,\mathrm{d}t$$

Goniometric Functions - Basic Formulae

$\sin^2\alpha+\cos^2\alpha=1.$

$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\sin\beta\cos\alpha \qquad \sin(2\alpha)=2\sin\alpha\cos\alpha.$

$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta \qquad \cos(2\alpha)=\cos^2\alpha-\sin^2\alpha.$

$\cos^2\alpha=\dfrac{1+\cos 2\alpha}{2}, \qquad \sin^2\alpha=\dfrac{1-\cos 2\alpha}{2}.$

A) $\sin\alpha+\sin\beta=2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2},$ B) $\sin\alpha-\sin\beta=2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2},$

C) $\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2},$ D) $\cos\alpha-\cos\beta=-2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}.$

Hyperbolic Functions – Basic Formulae

$\sinh x=\dfrac{e^x-e^{-x}}{2} \qquad \cosh x=\dfrac{e^x+e^{-x}}{2} \qquad \mathrm{tgh}\, x=\dfrac{e^x-e^{-x}}{e^x+e^{-x}} \qquad \mathrm{cotgh}\, x=\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$

$\sinh x+\cosh x=e^x \qquad \sinh x-\cosh x=-e^{-x} \qquad \cosh^2 x-\sinh^2 x=1$

Substitution in Integrals with Goniometric Functions

a) Odd power in denominator of a fraction: m odd $\Rightarrow m+1=2k$

$$\int\frac{\sin^n x}{\cos^m x}\,\mathrm{d}x=\int\frac{\sin^n x}{\cos^m x}\frac{\cos x}{\cos x}\,\mathrm{d}x=\int\frac{\sin^n x}{\cos^{2k} x}\cos x\,\mathrm{d}x=\int\frac{\sin^n x}{\left(1-\sin^2 x\right)^k}\cos x\,\mathrm{d}x=\begin{vmatrix}\sin x=t\\ \ldots..\end{vmatrix}\ldots$$

Intervaly: $(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi),\ k$ whole number

b) Odd power in numerator of a fraction: n odd $\Rightarrow n-1=2k$

$$\int\frac{\sin^n x}{\cos^m x}\,\mathrm{d}x=\int\frac{\sin^{n-1} x}{\cos^m x}\sin x\,\mathrm{d}x=\int\frac{\sin^{2k} x}{\cos^m x}\sin x\,\mathrm{d}x=\int\frac{\left(1-\cos^2 x\right)^k}{\cos^m x}\sin x\,\mathrm{d}x=\left|\cos x=t\right|\ldots$$

Intervaly: $(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi),\ k$ whole number

c) Substitution in integrals of type $\int R\left(\sin^2 x,\cos^2 x\right)\mathrm{d}x$

$\boxed{\mathrm{tg}\, x=t} \qquad x=\mathrm{arctg}\, t \qquad \dfrac{1}{\cos^2 x}\,\mathrm{d}x=\mathrm{d}t \qquad \mathrm{d}x=\dfrac{1}{t^2+1}\,\mathrm{d}t$

$\sin^2 x=\dfrac{t^2}{t^2+1}, \qquad \cos^2 x=\dfrac{1}{t^2+1}$

d) Substitution in Integrals of Type $\int R(\sin x,\cos x)\mathrm{d}x$

$\boxed{tg\,\frac{x}{2}=t} \qquad x=2\,\mathrm{arctg}\, t \qquad \dfrac{1}{2\cos^2\frac{x}{2}}\,\mathrm{d}x=\mathrm{d}t \qquad \mathrm{d}x=\dfrac{2}{1+t^2}\,\mathrm{d}t$

$\cos^2\frac{x}{2}=\dfrac{1}{1+t^2} \qquad \sin^2\frac{x}{2}=\dfrac{t^2}{1+t^2} \qquad \cos x=\dfrac{1-t^2}{1+t^2} \qquad \sin x=\dfrac{2t}{1+t^2}$

Substitution in Integrals with Roots

$\int R\left(x,\sqrt[s]{\dfrac{ax+b}{cx+d}}\right)\mathrm{d}x,$ upon condition $ad-cb\neq 0,$ substitution $\sqrt[s]{\dfrac{ax+b}{cx+d}}=t$

Laplace Transform

$$F(p) = \mathscr{L}\{f(t)\} = \int_0^\infty f(t)\,\mathrm{e}^{-pt}\,\mathrm{d}t$$

Laplace Transform of Some Basic Functions

$\mathscr{L}\{1\} = \frac{1}{p}$ Re $p > 0$

$\mathscr{L}\{t\} = \frac{1}{p^2}$ Re $p > 0$

$\mathscr{L}\{t^2\} = \frac{2}{p^3}$ Re $p > 0$

$\mathscr{L}\{t^n\} = \frac{n!}{p^{n+1}}$ Re $p > 0, n \in \mathbf{N}$

$\mathscr{L}\{e^{at}\} = \frac{1}{p-a}$ Re $p >$ Re a

$\mathscr{L}\{\sin(\omega t)\} = \frac{\omega}{p^2+\omega^2}$ Re $p > 0, \omega \in \mathbf{R}$

$\mathscr{L}\{\cos(\omega t)\} = \frac{p}{p^2+\omega^2}$ Re $p > 0, \omega \in \mathbf{R}$

Basic Rules and Properties of Laplace Transform

Linearity $\mathscr{L}\left\{\sum_{i=1}^{n} a_i f_i(t)\right\} = \sum_{i=1}^{n} a_i F_i(p)$

Shift $\mathscr{L}\{\mathrm{e}^{at} f(t)\} = F(p-a)$

Differentiation $\mathscr{L}\{t \cdot f(t)\} = -\frac{\mathrm{d}}{\mathrm{d}p}\big(F(p)\big)$

Scale change $\mathscr{L}\{f(t)\} = F(p) \Rightarrow \mathscr{L}\{f(kt)\} = \frac{1}{k}F\left(\frac{p}{k}\right)$

Integration $\mathscr{L}\left\{\frac{1}{t} f(t)\right\} = \int_p^\infty F(q)\,\mathrm{d}q$

L-transform of integral $\mathscr{L}\left\{\int_0^t f(z)\,\mathrm{d}z\right\} = \frac{F(p)}{p}$

Convolution $(f*g)(t) = (g*f)(t) = \int_0^t f(t-u)\cdot g(u)\,\mathrm{d}u = \int_0^t f(t)\cdot g(t-u)\,\mathrm{d}u$

L-transform of convolution $\mathscr{L}\{(f*g)(t)\} = F(p)\cdot G(p)$

Laplace Transform of Derivatives – for Solving Differential Equations

L-transform of $y(t)$ $\mathscr{L}\{y(t)\} = Y(p)$

L-transform of 1^{st} derivative $\mathscr{L}\{y'(t)\} = pY(p) - y(0_+)$

L-transform of 2^{nd} derivative $\mathscr{L}\{y''(t)\} = p^2Y(p) - p\cdot y(0_+) - y'(0_+)$

L-transform of n^{th} derivative $\mathscr{L}\{y^{(n)}(t)\} = p^nY(p) - [p^{n-1}\cdot y^{(0)}(0_+) + \ldots + py^{(n-2)}(0_+) + y^{(n-1)}(0_+)]$

Z-transform

$$\mathbf{Z}\left(\{f_n\}_{n=0}^{\infty}\right) = F(z) = \sum_{n=0}^{\infty} \frac{f_n}{z^n}, \quad \text{Notation}: \{f_n\}_{n=0}^{\infty} \triangleq F(z)$$

Z-transform of some Basic Sequences

Subject	**Z-transform**	
$\{f_n\}_{n=0}^{\infty} = \{1,0,0,0,\ldots\}$	1	
$\{\mathbf{1}\}_{n=0}^{\infty} = \{1,1,1,1,1,\ldots\}$	$\frac{z}{z-1}$	
$\{(-1)^n\}_{n=0}^{\infty} = \{1,-1,1,-1,\ldots\}$	$\frac{z}{z+1}$	
$\{n\}_{n=0}^{\infty} = \{0,1,2,3,4,\ldots\}$	$\frac{z}{(z-1)^2}$	
$\{n^2\}_{n=0}^{\infty} = \{0,1,4,9,16,\ldots\}$	$\frac{z(z+1)}{(z-1)^3}$	
$\{a^n\}_{n=0}^{\infty} = \{1,a,a^2,a^3,a^4,\ldots\}$	$\frac{z}{z-a}$	
$\{a^{n+1}\}_{n=0}^{\infty} = \{a,a^2,a^3,a^4,a^5\ldots\}$	$\frac{za}{z-a}$	(one step advance)
$\{a^{n-1}\}_{n=0}^{\infty} = \{0,1,a,a^2,a^3,\ldots\}$	$\frac{1}{z-a}$	(single delay)
$\{n \cdot a^n\}_{n=0}^{\infty} = \{0,a,2\cdot a^2,3\cdot a^3,\ldots\}$	$\frac{za}{(z-a)^2}$	(differentiation)
$\{n^2 \cdot a^n\}_{n=0}^{\infty} = \{0,a,4\cdot a^2,9\cdot a^3,\ldots\}$	$\frac{za(z+a)}{(z-a)^3}$	(differentiation)

Rules and Properties

Linearity	$\{a\cdot f_n + b\cdot g_n\}_{n=0}^{\infty} \triangleq a\cdot F(z) + b\cdot G(z)$
Frequency Scale	$\{a^n \cdot f_n\}_{n=0}^{\infty} \triangleq F\left(\frac{z}{a}\right)$
Time Delayed Shift	$\{f_{n-k}\}_{n=0}^{\infty} \triangleq z^{-k}\cdot F(z)$
Time Advance	$\{f_{n+k}\}_{n=0}^{\infty} \triangleq z^{k}\cdot \left(F(z) - \sum_{n=0}^{k-1} f_n z^{-n}\right)$
Differentiation	$\{n\cdot f_n\}_{n=0}^{\infty} \triangleq -z\cdot \frac{\mathrm{d}F(z)}{\mathrm{d}z}$
Complex Translation	$\{\mathrm{e}^{\lambda\cdot n}\cdot f_n\}_{n=0}^{\infty} \triangleq F(z\,\mathrm{e}^{\lambda})$

Rules for Solving Difference Equations

1^{st} order Forward Difference	$\{\Delta f_n\}_{n=0}^{\infty} \triangleq (z-1)\cdot F(z) - f_0 z$
k^{th} order Forward Difference	$\{\Delta^k f_n\}_{n=0}^{\infty} \triangleq (z-1)^k \cdot F(z) - z\sum_{i=0}^{k-1}(z-1)^{k-i-1}\Delta^i f_0$